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High Precision Test of QCD at Beijing Electron Positron Collider *

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Abstract

The generalized Crewther relation relates the cross section ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in e^+e^- annihilation with the Bjorken sum rule or the Gross-Llewellyn Smith sum rule in deep inelastic scattering and provides a fundamental connection for observables in Quantum Chromodynamics (QCD) without scale or scheme ambiguities. The ratio R can be measured at the upgrated Bejing Electron Positron Collider or the τ-Charm factory with higher precision and thus can be served for a high precision test of QCD in the Standard Model.

Key words: generalized Crewther relation, test of QCD, electron positron collider, deep inelastic scattering.

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One of the obstacles to test the Standard Model to high precision is the fact that perturbative predictions depend on the choice of renormalization scale and scheme. The situation is further complicated by the fact that computations in different sectors of the Standard Model are carried out using different renormalization schemes. One of the most illustrative examples is the recent observation of the high E_T jets by CDF collaboration at Tevatron [1]. The jet cross section calculated at NLO in $\overline{\mathrm{MS}}$ scheme using CTEQ3M parton distributions [2] fails to match the high E_T CDF data and this could suggest new physics for the substructure of the quark at high energy scale beyond the Standard Model [3]. However, it has been noticed that the jet cross section calculated at NLO in DIS scheme using CTEQ3D parton distributions matches pretty well the high E_T CDF data [4]. This introduces uncertainties about whether the CDF observation represents new physics signal or not.

Therefore fundamental relations in QCD with no scale or scheme ambiguities will be important for clean test of the Standard Model with high precision. There have been significant progress in theoretical investigations along this direction [5, 6, 7, 8]. The generalized Crewther relation [6, 7, 8] is such a relation connects the observables in e^+e^- annihilation and deep inelastic scattering.

The original Crewther relation [9] has the form

$$3S = KR' \tag{1}$$

where S is the value of the anomaly controlling $\pi^0 \to \gamma\gamma$ decay, K is the value of the Bjorken sum rule in polarized deep inelastic scattering, and R' is the isovector part of the annihilation cross section ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. Since S is unaffected by QCD radiative corrections, the Crewther relation requires that the QCD radiative corrections to $R_{e^+e^-}$ exactly cancel the radiation corrections to the Bjorken sum rule order by order in perturbative theory. The above Crewther relation is only valid in the case of conformally-invariant gauge theory, i.e., when the coupling α_s is scale invariant.

It is possible to express the entire radiative corrections to the annihilation cross

section as the "effective charge" $\alpha_R(\sqrt{s})$:

$$R_{e^+e^-}(s) \equiv 3\sum_f Q_f^2[1+\frac{\alpha_R(\sqrt{s})}{\pi}]. \tag{2}$$

Similarly, we can define the entire radiative correction to the Bjorken sum rule [10] as the effective charge α_{g_1} where Q is the lepton momentum transfer:

$$\int_0^1 \mathrm{d}x[g_1^{ep}(x,Q^2)-g_1^{en}(x,Q^2)] \equiv \frac{1}{6}\left|\frac{g_A}{g_V}\right|[1-\frac{\alpha_{g_1}(Q)}{\pi}]. \tag{3}$$

For non-conformally invariant gauge theory, the Crewther relation has been extended [6, 7, 8] to a generalized form:

$$(1+\hat{\alpha}_R)(1-\hat{\alpha}_{g_1})=1, \tag{4}$$

where $\hat{\alpha}_R = \frac{3C_F}{4\pi}\alpha_R$ and $\hat{\alpha}_{g_1} = \frac{3C_F}{4\pi}\alpha_{g_1}$, with $C_F = 4/3$. The scales s and Q^2 are connected through the formula:

$$\begin{aligned}\ln(\frac{Q^2}{s}) \quad &= \quad -\frac{7}{2}+4\zeta(3)-\left(\frac{\alpha_R(\sqrt{s})}{4\pi}\right)[(\frac{11}{12}+\frac{56}{3}\zeta(3)-16\zeta^2(3)-\frac{\pi^2}{3})\beta_0 \\ &\quad +\frac{26}{9}C_A-\frac{8}{3}C_A\zeta(3)-\frac{145}{18}C_F-\frac{184}{3}C_F\zeta(3)+80C_F\zeta(5)]\end{aligned} \tag{5}$$

where in QCD $C_A = 3$. We can also write down the analogous equation for the Bjorken sum rule by the Gross-Llewellyn Smith sum rule [11], defined as

$$\frac{1}{2}\int_0^1 \mathrm{d}x[F_3^{\nu p}(x,Q^2)+F_3^{\overline{\nu}p}(x,Q^2)] \equiv 3[1-\frac{\alpha_{GLS}(Q)}{\pi}] \tag{6}$$

and replace $\hat{\alpha}_{g_1}$ in Eq. (4) by $\hat{\alpha}_{GLS} = \frac{3C_F}{4\pi}\alpha_{GLS}$.

The experimental measurements of the R-ratio above the thresholds for the production of $c\overline{c}$-bound states, together with the theoretical fit performed in Ref. [12], provide the constraint

$$\frac{1}{3\sum_f Q_f^2}R_{e^+e^-}(\sqrt{s}=5.0\mathrm{GeV}) \simeq \frac{3}{10}(3.6\pm 0.1) = 1.08 \pm 0.03 \tag{7}$$

and thus

$$\frac{\alpha_R^{exp}(\sqrt{s}=5.0\mathrm{GeV})}{\pi} \simeq 0.08 \pm 0.03. \tag{8}$$

The corresponding expression for the effective coupling constants, when fitted with the generalized Crewther relation with some additional corrections [8] also taken into account, has the form

$$\frac{\alpha_{g_1}^{fit}(Q=12.33\pm1.20\mathrm{GeV})}{\pi}\simeq\frac{\alpha_{GLS}^{fit}(Q=12.33\pm1.20\mathrm{GeV})}{\pi}\simeq0.074\pm0.026. \quad (9)$$

The recent measurements for the Gross-Llewellyn Smith sum rule performed only at relatively small value of Q^2 [13]; however, one can use the results of the theoretical extrapolation [14] of the experimental data [15] and turn to the domain of large value of Q^2. Thus it is not difficult to extract the value for $\frac{\alpha_{GLS}}{\pi}$ from Ref.[14]:

$$\frac{\alpha_{GLS}^{extrapol}(Q=12.33\pm1.20\mathrm{GeV})}{\pi}\simeq0.093\pm0.042. \quad (10)$$

This interval overlaps with the result in Eq. (9) and this gives the empirical support for the generalized Crewther relation.

We notice that the precision of R in Eq .(7) is 3%, which is the precision can be reached by the available Beijing Electron Positron Collider within expected period. Therefore the above estimation in the precision test of the Standard Model by the generalized Crewther relation cannot be improved very much within some short period. There is no definite requirement of the precision of the data. Higher precision of data only improve the precision of the test and constrain further the magnitude for the new physics beyond Standard Model. But it can be reasonably expected that it will be difficult to find evidence for the breaking of the Standard Model if precision higher than 1% for R and 5% for the Gross-Llewellyn Smith sum cannot be reached. In this case, Eq. (9) will be changed to

$$\frac{\alpha_{g_1}^{fit}(Q=12.33\pm1.20\mathrm{GeV})}{\pi}\simeq\frac{\alpha_{GLS}^{fit}(Q=12.33\pm1.20\mathrm{GeV})}{\pi}\simeq0.074\pm0.008, \quad (11)$$

and Eq. (10) will be changed to

$$\frac{\alpha_{GLS}^{experiment}(Q=12.33\pm1.20\mathrm{GeV})}{\pi}\simeq0.093\pm0.006, \quad (12)$$

if the central values still keep unchanged. There will be no much difficulty to change the precision for Eq. (10) from the present 45% to 5% if direct experimental measurement at high Q^2 will be performed rather than by using theoretical extrapolation from relatively small Q^2 of the available experimental data.

It is worthwhile to point out that there might be non-perturbative or higher-twist contributions [16] to the generalized Crewther relation at small Q^2 and they could be estimated with theoretical progress. Thus it is possible to reduce uncertainties in the relation. In order to check the consequence of the generalized Crewther relation at a higher confidence level, it will be necessary to reduce the experimental error of the measurement of $R_{e^+e^-}$ at $\sqrt{s} \simeq 5$ GeV and to have more precision information on the value of the Gross-Llewellyn Smith sum rule at $Q^2 = 150$ GeV^2 or to measure the polarized Bjorken sum rule at this momentum transfer. The Bjorken and Gross-Llewellyn Smith sum rules are under measurements (or plan) by a number of collaborations at SLAC, CERN and DESY and data with high precision will be available in the future. The ratio $R_{e^+e^-}$ at $\sqrt{s} \simeq 5$ GeV can be attacked after the upgrade of the Beijing Electron Positron Collider or the operation of the future τ-charm factory. Supplied with further theoretical progress we expect to know more from the future measurement of the ratio $R_{e^+e^-}$ at the Beijing Electron Positron Collider (it's upgrade or τ-charm factory) and its role in the high precision test of QCD.

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